Paper 3 Revision

Key topics to practice for 11th June

All the topics listed below are likely to appear in some form in paper 2 or 3.

Scan the QR code for solutions and then select

Very Likely topics

Compound interest/Growth and decay	Volume of 3D shapes	Recurring decimals to fractions	Changing the subject
Product rule for counting	Pythagoras	Form algebraic equation in context	Iteration
Sine Rule	Speed/time graphs	Similar Area/Volume	Sequences
Use of probability to estimate/ work out an amount	Cumulative Frequency/Box plots	Increase/Decrease by %	

Likely topics

Gradients, Intercepts, y=mx+c	Simplifying algebraic expressions	Congruence	Expanding/factorising
Solve linear equations	Averages and range	Venn diagrams	

Very Likely – Increase/Decrease by a %

Q1.

A bus route had 90 000 passengers last year.
The number of passengers was predicted to increase

by 3% this year $\times 1.03$
and then
by 8% next year. $\times 1.08$

Is the predicted number of passengers for **next** year more than 100 000 ? **Yes**
You **must** show your working.

$$90\,000 \times 1.03 \times 1.08 = 100116$$

(Total 3 marks)

Q2.

Magana decides to put £500 into an account that pays compound interest.
She wants to have **at least** £560 in the account after 3 years.

Work out to 1 decimal place the **minimum** annual interest rate she needs. *Trial + Improvement*

4% $500 \times 1.04^3 = 562.432$
3.9% $500 \times 1.039^3 = 560.8111595 \rightarrow$ **3.9%**
3.8% $500 \times 1.038^3 = 559.193436$

(Total 3 marks)

Q3.

£2448 is invested in an account at a rate of compound interest.
One year after the investment there is £2496.96 in the account.

How much is in the account four years after the investment?

$$\frac{2496.96}{2448} = 1.02 \rightarrow 2\% \text{ interest}$$

$$2448 \times 1.02^4 = 2649.79$$

(Total 3 marks)

£2649.79
(nearest penny)

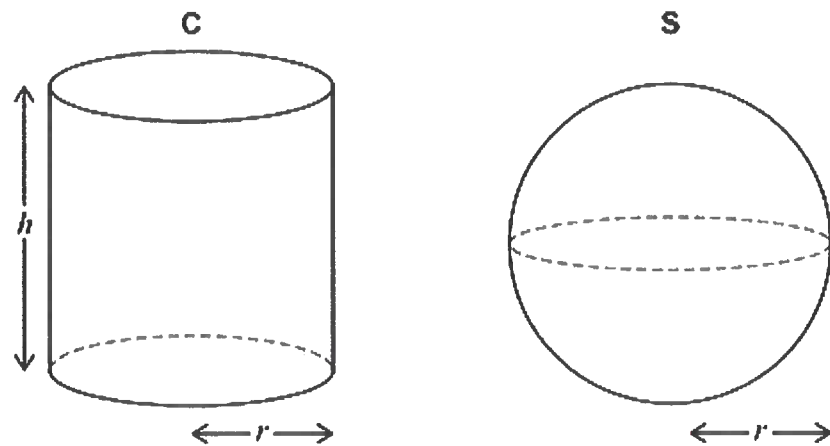## Very Likely – Volume of 3D shapes

Q1.

A cylinder, C, and a sphere, S, each have radius r

C has height h

Volume of a sphere = $\frac{4}{3}\pi r^3$

where r is the radius

(a)   volume of C = volume of S

Work out the ratio $r : h$

You **must** show your working.

$\underline{C}$ $Vol = \pi r^2 h$

$\underline{S}$ $Vol = \frac{4}{3}\pi r^3$

$Vol \, of \, C = Vol \, of \, S$

$$\pi r^2 h = \frac{4}{3}\pi r^3$$

$\div \pi$ $\times 3$ $\div r^2$ $\Big|$ $r^2 h = \frac{4}{3}r^3$ $\Big| \div \pi$ $\times 3$

$$3r^2 h = 4r^3$$

$$3h = 4r$$

$\underline{Careful}$ $r:h \neq 4:3$ ✗

$\underline{r:h = 3:4}$ ✓

(3)

(b) A **different cylinder** has radius $3r$ and height $2h$.

How many times bigger is the volume of this cylinder than the volume of C?

(2)
(Total 5 marks)

Vol of new Cylinder $= \pi(3r)^2(2h)$

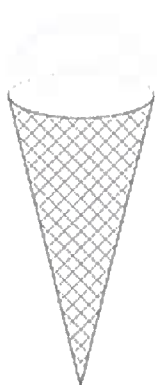$= \pi \times 3r \times 3r \times 2h$

$= 18\pi r^2 h$

Vol of C $= \pi r^2 h$

Sol new Cylinder is <u>18x</u> bigger

Q2.

Outside a cafe there is a large plastic ice cream cornet.

The cornet is a hemisphere on top of a cone.

The cone and the hemisphere each have radius 24 cm

The cone has perpendicular height 117 cm

Volume of a cone = $\dfrac{1}{3}\pi r^2 h$

r is the radius

h is the perpendicular height

Volume of a hemisphere = $\dfrac{2}{3}\pi r^3$

r is the radius

(a)   Work out the total volume of the cornet.

$\rightarrow$ *Type it in like this!*

Cone $Vol = \frac{1}{3} \times \pi \times 24^2 \times 117$

$= 22464\pi$

Hemisphere

$Vol = \frac{2}{3} \times \pi \times 24^3$

$= 9216\pi$

$\Big\} +$ $= 31680\pi$

$= 99525 \cdot 7 \text{ cm}^3$

(4)

(b)   The actual cornets that the cafe sells are **similar** to the plastic one.   *Similar $\rightarrow$ enlargement*

For the actual cornets, the cone and the hemisphere each have radius 2 cm

How many times greater is the volume of the plastic cornet than an actual cornet?

Length Scale factor $= \frac{24}{2} = 12$
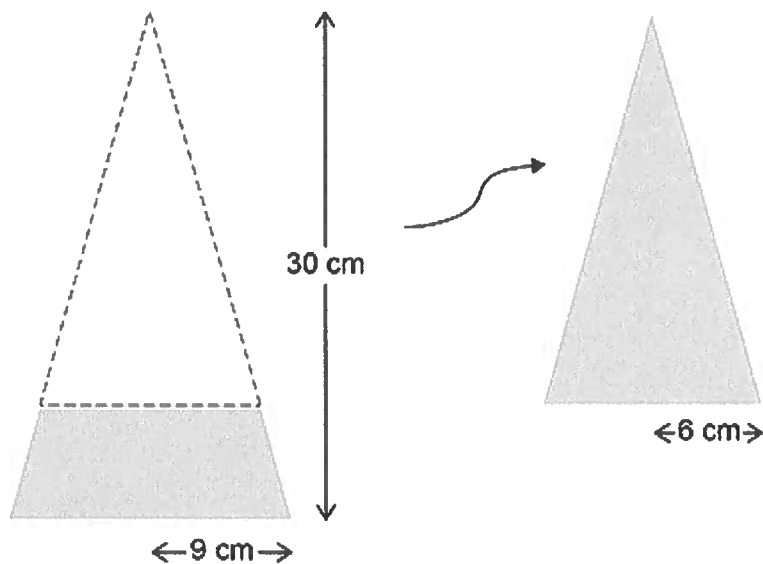
Vol Scale factor $= 12^3 = 1728$

times greater.

Q3.

Alec makes a bowl for dog food from a solid wooden cone.
The sketches show how the bowl is made.

The cone has radius 9 cm and perpendicular height 30 cm
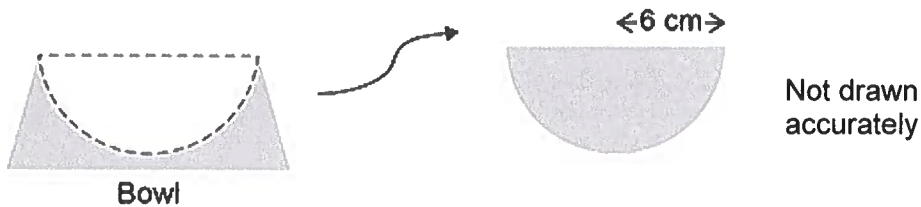A smaller cone, with radius 6 cm, is removed.

30 cm

Not drawn
accurately

←6 cm→

←9 cm→

Volume of a cone = $\frac{1}{3}\pi r^2 h$

where r is the radius and h is the perpendicular height

A hemisphere with radius 6 cm is then removed.

Not drawn accurately

Bowl

$$\text{Volume of a hemisphere} = \frac{2}{3}\pi r^3 \quad \text{where } r \text{ is the radius}$$

Work out the volume of the remaining wood that forms the bowl.

Big Cone $\quad \text{Vol} = \frac{1}{3} \times \pi \times 9^2 \times 30 = 810\pi$

Small Cone $\quad \text{Vol} = \frac{1}{3} \times \pi \times 6^2 \times \boxed{20} = 240\pi$

$\uparrow$
?

The two cones are similar so the height and radius are in proportion.

$9 \xrightarrow[\times \frac{2}{3}]{} 6$

$30 \xrightarrow[\times \frac{2}{3}]{} 20$

$\square = 810\pi - 240\pi = 570\pi$

Hemisphere $= \frac{2}{3} \times \pi \times 6^3 = 144\pi$

$\square = 570\pi - 144\pi = 626\pi$
$= 1966.6 \text{ cm}^3$

(Total 5 marks)

Very Likely – Recurring decimals to fractions

Q1.

Which of these when converted to decimals are recurring decimals?
Circle your answers.

$$\boxed{\frac{1}{3}} \qquad \pi \quad \times \quad \sqrt{3} \qquad \frac{3}{16} \qquad \boxed{\frac{5}{7}}$$

no recurring pattern $\times$

$0.1875 \times$

$0.\overline{714285}$ **(Total 2 marks)**

Q2.

Work out $\quad 0.70\dot{4}\dot{8} - 0.001$

Use $\Box$ button $= 0.703848484\ldots$

$= 0.703\dot{8}\dot{4}$

Circle your answer.

$0.70\dot{3}\dot{8} \qquad 0.703\dot{8} \qquad 0.703\dot{8}\dot{3} \qquad \boxed{0.703\dot{8}\dot{4}}$

(Total 1 mark)

Q3.

Prove algebraically that $\quad 2.7\dot{5} \quad$ converts to the fraction $\dfrac{124}{45}$

let $x = 2.7555\ldots$

$$10x = 27.5555\ldots$$
$$-\quad x = 2.7555\ldots$$
$$9x = 24.8000$$

$$9x = 24.8$$

$$x = \frac{24.8}{9}$$

$$\frac{24.8}{9} \times \frac{5}{5} = \frac{124}{45}$$

(Total 3 marks)

Very Likely – Changing the subject

Q1.

$a \times b^4 = c$

$$\div b^4 \quad \left| \begin{array}{l} a \times b^4 = c \\ a = \dfrac{c}{b^4} \end{array} \right| \div b^4$$

Circle the correct expression for a.

$$\dfrac{c}{\sqrt[4]{b}} \qquad\qquad \dfrac{c}{b^{-4}} \qquad\qquad \left(\dfrac{c}{b}\right)^4 \qquad\qquad \boxed{\dfrac{c}{b^4}}$$

(Total 1 mark)

Q2.

Rearrange $\quad a = \dfrac{b}{c} + 5 \quad$ to make c the subject.

$$
\begin{array}{r}
-5 \\
\times c \\
\div(a-5)
\end{array}
\left|
\begin{array}{l}
a = \dfrac{b}{c} + 5 \\[4pt]
a - 5 = \dfrac{b}{c} \\[4pt]
c(a-5) = b \\[4pt]
c = \dfrac{b}{(a-5)}
\end{array}
\right|
\begin{array}{l}
-5 \\
\times c \\
\div(a-5)
\end{array}
$$

(Total 3 marks)

Q3.

Rearrange $\quad 9m + 4(2m - 1) = p^2 + pm \quad$ to make m the subject.

$$9m + 8m - 4 = p^2 + pm$$

$$
\begin{array}{r}
-pm \\
+4 \\
\\
\text{factorise} \\
\\
\div(17-p)
\end{array}
\left|
\begin{array}{l}
17m - 4 = p^2 + pm \\
17m - 4 - pm = p^2 \\
17m - pm = p^2 + 4 \\
m(17 - p) = p^2 + 4 \\
m = \dfrac{p^2 + 4}{17 - p}
\end{array}
\right.
\begin{array}{l}
-pm \\
+4 \\
\\
\\
\div(17-p)
\end{array}
$$

$$\left(or \quad m = \dfrac{-p^2 - 4}{p - 17} \right)$$

(Total 4 marks)

Q4.

Rearrange $p = \dfrac{2m+1}{1-m}$ to make m the subject.

$$p = \frac{2m+1}{1-m}$$

$\times(1-m)$ ⎪ ⎪ $\times(1-m)$

$$p(1-m) = 2m+1$$

$$p - pm = 2m+1$$

$+pm$ ⎪ $p = 2m+1+pm$ ⎪ $+pm$

-1 ⎪ $p-1 = 2m+pm$

$$p-1 = m(2+p)$$ factorise

$\div(2+p)$ ⎪ $\dfrac{p-1}{(2+p)} = m$ ⎪ $\div(2+p)$

$$m = \frac{p-1}{2+p}$$

(Total 4 marks)

Very Likely – Product rule for counting

Q1.

A vending machine has a different item in each section.
It sells

7 drinks, 3 of which are juice

5 snacks, 2 of which are fruit bars

11 meals, 4 of which are salad.

One drink, one snack and one meal are chosen at random.

Show that the probability of getting a juice, a fruit bar and a salad is **more** than 5%

Total combinations = $7 \times 5 \times 11 = 385$

Total combinations of Juice, fruit bars and salad
$= 3 \times 2 \times 4 = 24$

Probability $= \dfrac{24}{385} = 0.0623376$
$\times 100$

$= 6.233766\%$

$> 5\%$

(Total 3 marks)

Q2.

Fatima is choosing a 4-digit code.

Each digit is a whole number from 0 to 9

She decides

    all her digits will be odd numbers

    no digits will be repeated.

How many different codes can she make?

5 possible numbers   4 possible numbers   3 possible numbers   2 possibilities

1, 3, 5, 7, 9

$$5 \times 4 \times 3 \times 2 = 120$$

(Total 2 marks)

Q3.

A code has 4 digits.

    Each digit is a number from 0 to 9

    Digits may be repeated.

The code starts   5 4 1

5	4	1	

(a)   Amy knows the last digit is odd but **not** 7

        She chooses a different odd number at random.

→ 1, 3, 5, ✗, 9

        What is the probability that she chooses the correct number?

$\frac{1}{4}$ (or 0·25 or 25%)

(One out of the 4 numbers will be correct)

(1)

(b) The 4-digit code is changed to an even number. → *must end in* $0, 2, 4, 6$ *or* 8
   The first digit is 3   *to be even.*

   How many possible codes are there?

$3 \quad \underline{0-9} \quad \underline{0-9} \quad \underline{}$
$\qquad \uparrow \qquad \uparrow \qquad \uparrow$
$0,2,4,6,8$

10 possible numbers   10 possible numbers   5 possible numbers

$$10 \times 10 \times 5 = 500$$

(2)
(Total 3 marks)

Very Likely – Pythagoras

Q1.

Use Pythagoras' theorem to work out the value of x.

17.2 cm

x

12.9 cm
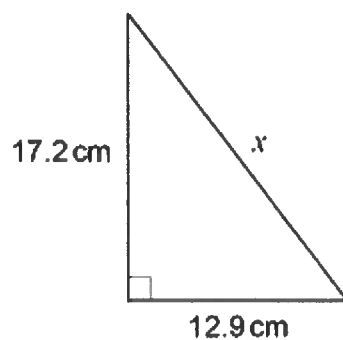
Not drawn accurately

Can enter it in to Calculator in one go.

$$\sqrt{12.9^2 + 17.2^2}$$

$$= 21.5 \, cm$$

(Total 3 marks)

Q2.

A shape is made by joining a right-angled triangle to a rectangle.

30 cm   16 cm

Not drawn
accurately

52 cm
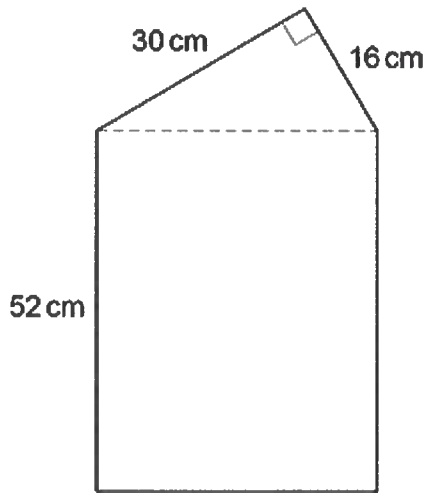
Work out the area of the shape.

- Area of triangle $= \dfrac{16 \times 30}{2} = 240 \text{ cm}^2$

- Use pythag to get width of rectangle

$$\sqrt{16^2 + 30^2} = 34$$

- Area of rectangle $= 52 \times 34 = 1768$

- Total area $= 240 + 1768 = \underline{\underline{2008}} \text{ cm}^2$

Q3.

All dimensions are in centimetres.

Not drawn accurately

10y

x

7x
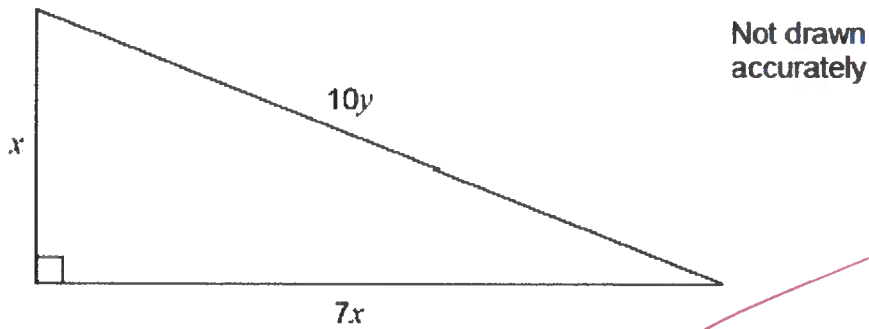
write as a surd.

Use Pythagoras' theorem to work out the (exact value) of $\dfrac{x}{y}$

must use brackets

$$x^2 + (7x)^2 = (10y)^2$$

$$= x^2 + 49x^2 = 100y^2$$

$$\begin{array}{ll} 7x \times 7x = 49x^2 \\ 10y \times 10y = 100y^2 \end{array}$$

$\div 50$
$\div y^2$
$\sqrt{}$

$$50x^2 = 100y^2$$

$$x^2 = 2y^2$$

$$\frac{x^2}{y^2} = 2$$

$$\sqrt{\frac{x^2}{y^2}} = \sqrt{2}$$

$$\frac{x}{y} = \sqrt{2}$$

$\div 50$
$\div y^2$
$\sqrt{}$

(Total 3 marks)

Very Likely – Forming algebraic equation in context

Q1.

AB, *CD* and *EF* are straight lines.

F

Not drawn
accurately

$2x + 10°$

B

w

A

70

D

$3x - 20°$

y

C

E
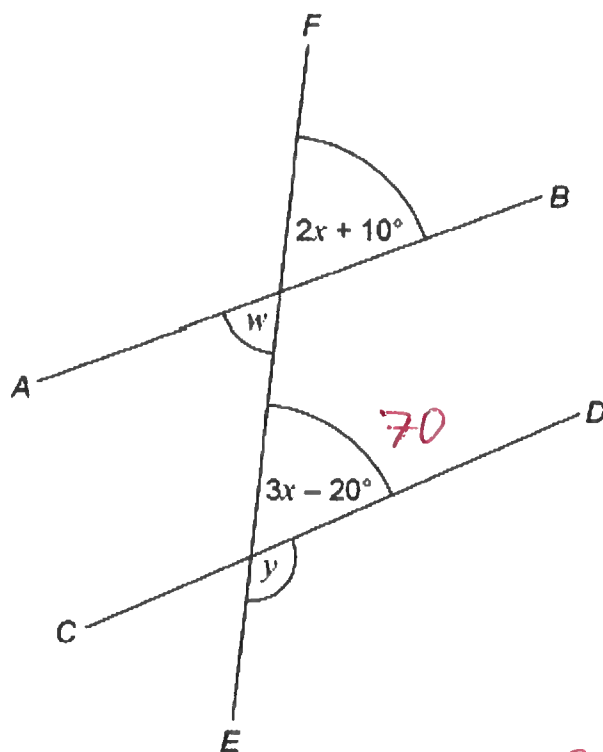
$\rightarrow 2x + 10 = 3x - 20$

(a) Ava assumes that *AB* and *CD* are parallel.

What answer should she get for the size of angle *y*?

$$2x + 10 = 3x - 20$$
$$-2x \quad \quad 10 = x - 20 \quad +20$$
$$+20 \quad \quad 30 = x$$

So $\quad 3x - 20 = 3 \times 30 - 20 = 70$

So $\quad y = 180 - 70 = 110°$

(4)

(b)   In fact,

AB and CD are **not** parallel

angle w is 60°

What effect does this have on the size of angle y?

Tick a box.

☑ y is bigger

☐ y is the same

☐ y is smaller

Show working to support your answer.

If $w = 60$ $2x + 10 = 60$ also

$$-10 \begin{vmatrix} 2x + 10 = 60 \\ 2x = 50 \\ x = 25 \end{vmatrix} -10 \\ \div 2$$

$\div 2$

If $x = 25$

$3x - 20 = 3 \times 25 - 20$

$= 55$

So $y = 180 - 55 = 125$

> 110

(3)

(Total 7 marks)

Q2.

The table shows information about some CDs.

Type	Rock	Pop	Jazz
Number of CDs	2	x	$2x + 5$

A CD is chosen at random.

The probability it is **rock** is $\dfrac{1}{20}$ $= \dfrac{2}{40}$ $\rightarrow$ ∴ 'Therefore' there must be 40 in total. (CDs)

Work out the probability it is jazz.

$$2 + x + 2x + 5 = 40$$

$$-7 \begin{vmatrix} 3x + 7 = 40 \\ 3x = 33 \\ x = 11 \end{vmatrix} \begin{array}{c} -7 \\ \div 3 \end{array}$$
$$\div 3$$

$$\text{Jazz CDs} = 2 \times 11 + 5$$
$$= 27$$

$$P(\text{Jazz}) = \frac{27}{40}$$

(Total 4 marks)

Very Likely – Iteration

Q1.

An approximate value of a root of an equation, x, can be found using the iterative formula

$$x_{n+1} = \sqrt[3]{5(x_n)^2 - 2x_n - 3}$$

The starting value is $x_1 = 4$

(a) Work out the values of x_2 and x_3

$$x_2 = \sqrt[3]{5(x_1)^2 - 2(x_1) - 3} = \sqrt[3]{5 \times 4^2 - 2 \times 4 - 3} = 4.10156593$$

$$x_3 = \sqrt[3]{5(4.101\ldots)^2 - 2 \times (4.101\ldots) - 3} = 4.177641648$$

best practice! Type $\boxed{4\ =}$ into calculator. then....

Type $\sqrt[3]{5 \times ANS^2 - 2 \times ANS - 3}$ Then press $\boxed{=}$ for x_2
Then press $\boxed{=}$ again for x_3

(2)

(b) By continuing the iteration, show that the value of x is more than 4.25

keep pressing $\boxed{=}$ until your answer does not change

$x_{65} = 4.388489747$

You can keep count! like I did

(1)
(Total 3 marks)

(a) Use this iterative process to find a solution to 4 decimal places of $x^2 - 7x - 3 = 0$

Start with the value $x = 7$

$$\frac{ANS^2 + 3}{2 \times ANS - 7}$$

for x_3

$$\frac{7.4286^2 + 3}{2 \times 7.4286 - 7}$$

etc...

$x_1 = 7$
$x_2 = 7.428571 \quad (7.4286)$
$x_3 = 7.40519\ldots \quad (7.4052)$
$x_4 = 7.40512\ldots \quad (7.4051)$
$x_5 = 7.40512\ldots \quad (7.4051)$

$$\underline{x = 7.4051}$$

(3)

(b) By substituting your answer to part (a) into $x^2 - 7x - 3$

comment on the accuracy of your solution to $x^2 - 7x - 3 = 0$

$$7.4051^2 - 7 \times 7.4051 - 3$$

$$= -0.00019399\ldots$$

This is a good estimate as it
is very close to 0.

(2)
(Total 5 marks)

Q2.

$$x_{n+1} = 5 - \frac{1}{x_n}$$

Use $x_1 = 1$ to work out an approximate solution to $x = 5 - \frac{1}{x}$
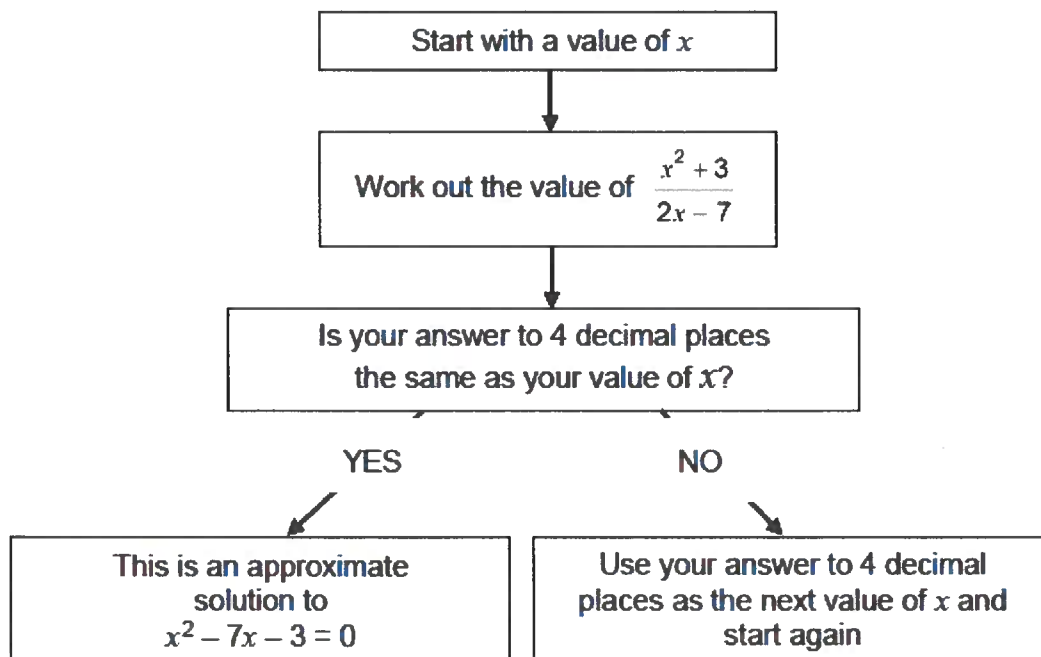
Give your answer to 4 significant figures.

Press ① = Then

Press $5 - \frac{1}{\text{ANS}}$ then = until the answer to 4 d.p 5f. does not change

$$x = \underline{4.791}$$

4 sf

(Total 3 marks)

Q3.

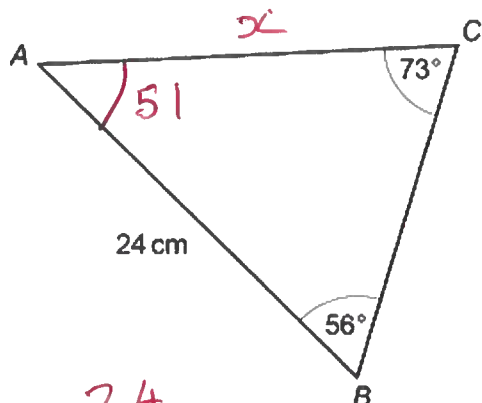This iterative process can be used to find approximate solutions to $x^2 - 7x - 3 = 0$

Start with a value of x

↓

Work out the value of $\dfrac{x^2 + 3}{2x - 7}$

↓

Is your answer to 4 decimal places the same as your value of x?

YES — This is an approximate solution to $x^2 - 7x - 3 = 0$

NO — Use your answer to 4 decimal places as the next value of x and start again

Very Likely – Sine rule

Q1.

Work out the area of triangle *ABC*.

$Area = \frac{1}{2}ab\sin C$

$(or \ \frac{1}{2}bc\sin A \ etc...)$

A — — — x — — — C

51 $73°$

$24\ cm$

Not drawn accurately

$56°$

B

$180 - (56 + 73)$
$= 51$

$\dfrac{x}{\sin 56} = \dfrac{24}{\sin 73}$

$x = \dfrac{24}{\sin 73} \times \sin 56 = 20.8060....$

$Area = \frac{1}{2} \times 24 \times 20.8060.... \times \sin 51$

$= 194.0 \ cm^2$

(Total 4 marks)

Q2.

$14\ cm$

x

$55°$

$30°$
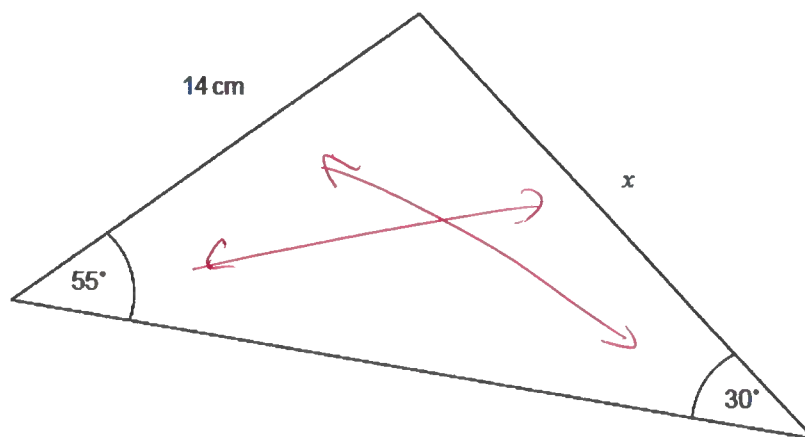
Not drawn accurately

Work out the value of x

$\dfrac{x}{\sin 55} = \dfrac{14}{\sin 30}$

$x = \dfrac{14}{\sin 30} \times \sin 55$

$= 22.9 \ cm$

(Total 2 marks)

Q3.

ABC and ACD are triangles.

B   20 cm   A

Not drawn accurately

35°

49°

C   16 cm   D

Work out the size of angle x.

Op

x

49

16

Adj

$\times 16$ | $\tan 49 = \dfrac{x}{16}$ | $\times 16$

$16 \times \tan(49) = x$

$= 18.405\ldots\ldots$

20

x

18.405....

35
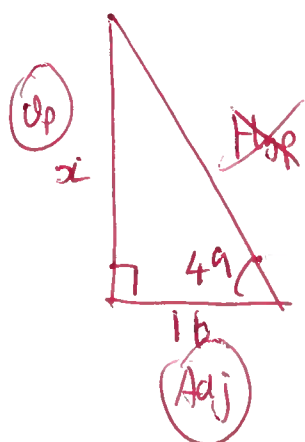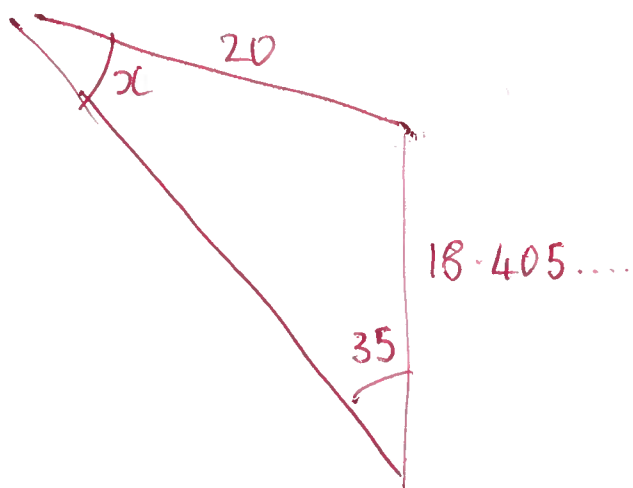
$\dfrac{\sin x}{18.405\ldots} = \dfrac{\sin 35}{20}$

$\therefore \sin x = \dfrac{\sin 35}{20} \times 18.405\ldots$

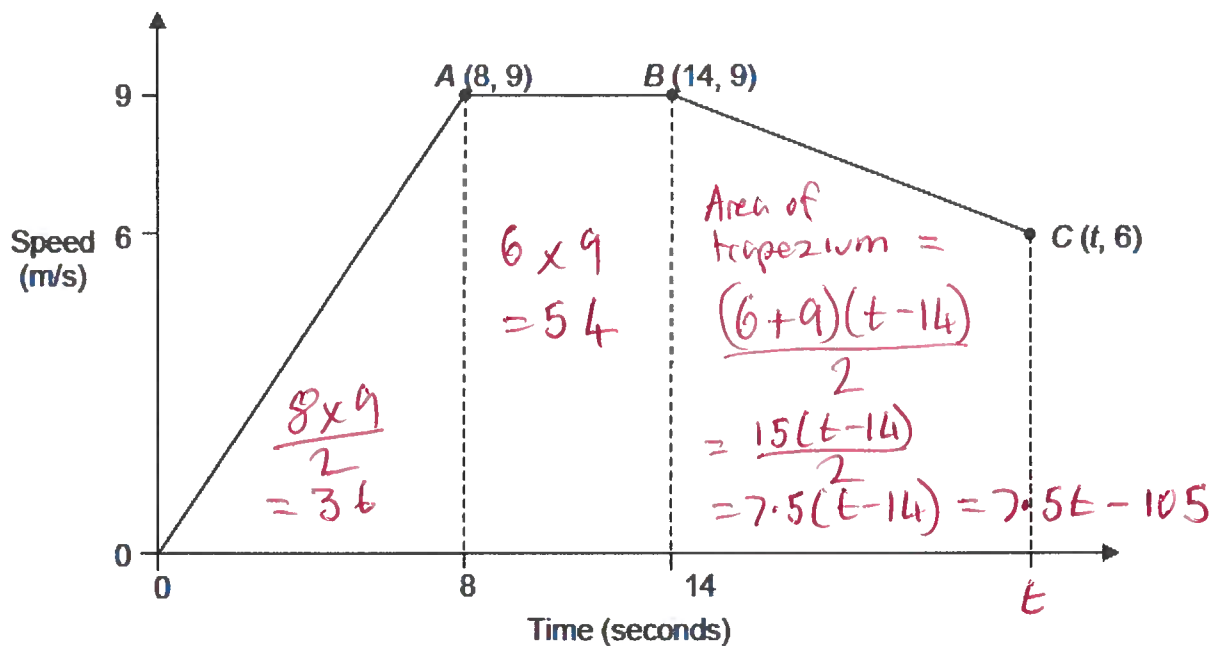$\sin x = 0.5278\ldots$

$x = \sin^{-1}(0.5278\ldots)$

$= 31.9^{0}$

(Total 5 marks)

(1dp)

Very Likely – Speed/time graphs

Q1.

Here is a sketch of a speed-time graph for part of a journey.

Speed (m/s) on vertical axis, values 9, 6, 0

A (8, 9)   B (14, 9)   C (t, 6)

Annotations on graph:

$\frac{8 \times 9}{2} = 36$

$6 \times 9 = 54$

Area of trapezium $= \frac{(6+9)(t-14)}{2} = \frac{15(t-14)}{2} = 7\cdot5(t-14) = 7\cdot5t - 105$

Time (seconds), axis marks at 8, 14, t

The average speed from 0 to t seconds was 7.2 m/s

Work out the value of t

(Total 5 marks)

Ave speed = $\frac{\text{Total distance}}{\text{total time}}$ (7.2)

total time $= t$
total distance = area under graph.

$\times t$ $\frac{36 + 54 + 7\cdot5t - 105}{t} = 7\cdot2$ $\times t$

$90 + 7\cdot5t - 105 = 7\cdot2t$

$-7\cdot2t$ $7\cdot5t - 15 = 7\cdot2t$ $-7\cdot2t$

$+15$ $0\cdot3t - 15 = 0$ $+15$

$\div 0\cdot3$ $0\cdot3t = 15$ $\div 0\cdot3$

$t = \frac{15}{0\cdot3}$

$t = 50$

Q2.

Here is the velocity-time graph of a cyclist for 40 seconds.
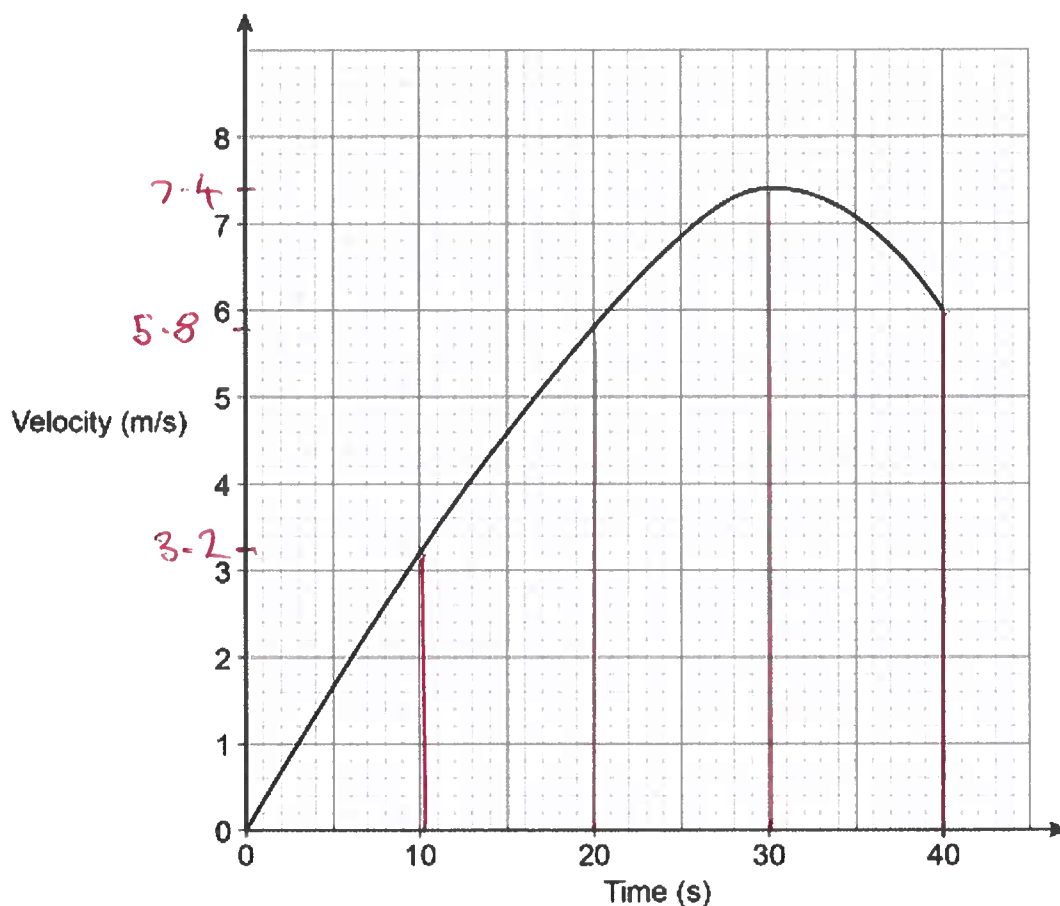
(a) By dividing the area under the graph into four sections of equal widths, estimate the distance travelled during the 40 seconds.

$$\frac{10 \times 3 \cdot 2}{2} + \frac{(3 \cdot 2 + 5 \cdot 8) \times 10}{2} + \frac{(5 \cdot 8 + 7 \cdot 4) \times 10}{2} + \frac{(7 \cdot 4 + 6) \times 10}{2}$$

$$= 16 + 45 + 66 + 67$$

$$= \underline{194\,m}$$

(3)

(b) Work out the average acceleration of the cyclist during the 40 seconds. State the units of your answer.

0 to 6 m/s in 40 seconds

$$\frac{6}{40} = 0.15 \text{ m/s}^2$$

(2)

(Total 5 marks)

Q3.

The speed-time graph for a car's journey is shown.
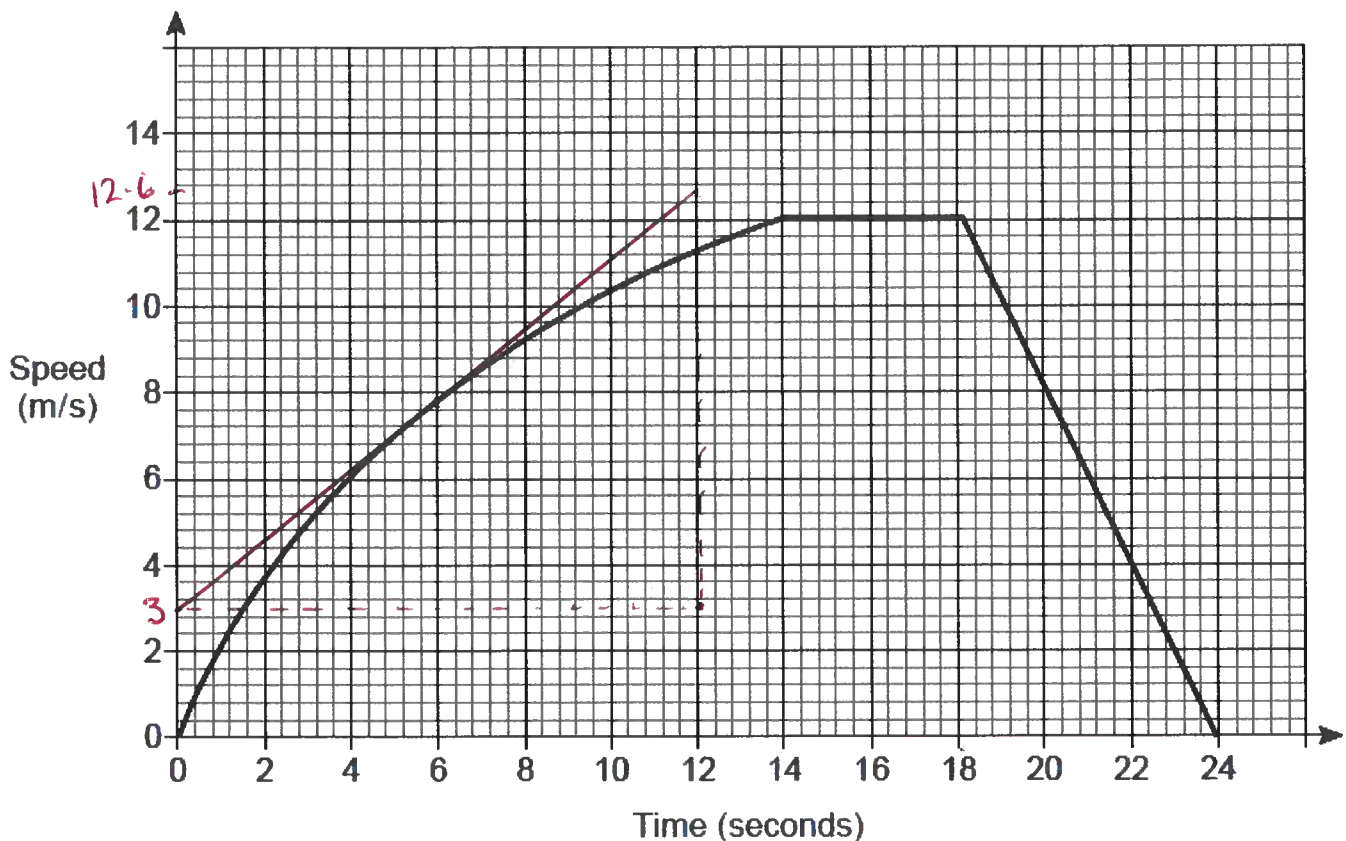
(a) Estimate the acceleration at 6 seconds. — Draw tangent to curve at $t = 6$
You **must** show your working.

Gradient = acceleration = $\dfrac{12 \cdot 6 - 3}{12} = 0 \cdot 8 \, m s^{-2}$

(3)

(b) (Estimate) the average speed of the car for the journey. $= \dfrac{Total \; distance}{Total \; time.}$
You **must** show your working.
← I will treat the area under the curve
    as a big triangle
[I could use trapezia]
   but only an estimate required.

$= \dfrac{\dfrac{14 \times 12}{2} + 4 \times 12 + \dfrac{6 \times 12}{2}}{24}$

(4)

(c) Evaluate your answer to part (b).

$= 7 \, m/s$

Tick a box.

☑ underestimate   ☐ exact   ☐ overestimate

Comment

The area under the curve is bigger than
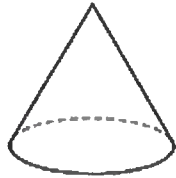the area I calculated.

(1)

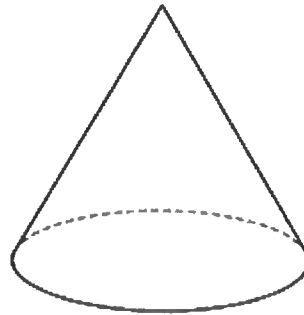**(Total 8 marks)**

Very Likely – Similar Area/Volume

Q1.   → Enlargements.

Here are two similar cones.

Cone A   Cone B

The surface area of cone A is 2 m²

The surface area of cone B is 4.5 m²

Work out the ratio   radius of cone A : radius of cone B

Give your answer in the form 1 : n

$4.5 \div 2 = 2.25$

= Area Scale factor

$\sqrt{2.25} = 1.5$
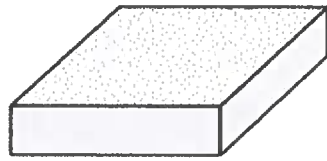
= Length Scale factor

radius of Cone A : radius of Cone B

=   1 : 1.5
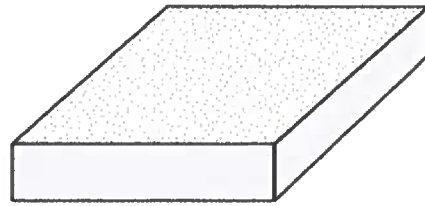
(Total 3 marks)

Q2.

Here are two square-based paving stones.

The stones are similar solids.

20 cm   25 cm

The price per cm³ is the same for both stones. → must work out volume SF

The price of the **larger** stone is £17.50

Work out the price of the smaller stone.

$$\frac{25}{20} = 1.25 = \text{Length Scale factor}$$

$$\text{Vol Scale factor} = 1.25^3$$

So £17.50 ÷ 1.25³

$$= £8.96$$

(Total 4 marks)

Very Likely – Sequences

Q1.

Work out the next term of this quadratic sequence.

5   8   14   23 $\boxed{35}$

$+3$ $+6$ $+9$ $+12$

(Total 2 marks)

Q2.

A is an **arithmetic** progression.
Here are the first four terms.

13   16   19   22

$+3$ $+3$

G is a **geometric** progression.
Here are the first four terms.

2   4   8   16

$\boxed{n\text{th term of A} = \text{8th term of G}}$

Work out the value of n.

n^{th} term of $A = 3n + 10$ [3 times table +10]

8th term of $G =$

$5^{th} \rightarrow 32 \; \times 2$
$6^{th} \rightarrow 64 \; \times 2$
$7^{th} \rightarrow 128 \; \times 2$
$8^{th} \rightarrow 256$

So $\begin{vmatrix} 3n + 10 = 256 \\ 3n = 246 \\ n = 82 \end{vmatrix}$

(Total 4 marks)

Q3.

Here are the first four terms of a quadratic sequence.

$$3 \qquad 20 \qquad 47 \qquad 84$$

Work out an expression for the nth term of the sequence.

$$3 \quad 20 \quad 47 \quad 84$$
$$17 \quad 27 \quad 37$$
$$10 \quad 10$$

$$10 \div 2 = 5$$

($5n^2$ will be part of nth term)

$$
\begin{array}{llllll}
5 & 3 & 20 & 47 & 84 \\
5n^2 & 5 & 20 & 45 & 80 \\
5-5n^2 & -2 & 0 & 2 & 4 \quad \rightarrow \text{This sequence is}
\end{array}
$$

$$2n - 4$$

So

$$+5n^2 \left| \begin{array}{l} 5 - 5n^2 = 2n - 4 \\[2mm] 5 \;\; = 2n - 4 + 5n^2 \end{array} \right| +5n^2$$

$$\text{or} \quad 5n^2 + 2n - 4$$

(Total 4 marks)

Q4.

(a) The nth term of a sequence is $n^2 + 12n + 27$

By factorising, or otherwise, show that the 20th term can be written as the product of two prime numbers.

$$(n+3)(n+9)$$

$$n = 20$$

$$(20+3)(20+9) = 23 \times 29$$

(2)

(b) The nth term of a different sequence is $n^2 - 6n + 14$

By completing the square, or otherwise, show that every term is positive.

$$(n-3)^2 - 3^2 + 14$$
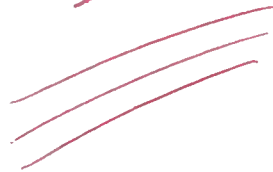
$$= (n-3)^2 - 9 + 14$$

$$= \underbrace{(n-3)^2 + 5}$$

This will always be ≥ 0
as squaring a number will never be
negative.

So

$n^2 - 6n + 14$
  is always ≥ 5

(3)

(Total 5 marks)

Very Likely – Use of probability to estimate/work out an amount

Q1.

Rosie makes phone calls to try to sell broadband.

Today, she made 120 calls.

The table shows the results.

Result of call	Frequency
Not answered	33
Answered but sale not made	81
Answered and sale made	6

(a) Write down the relative frequency that a call was **not answered**.

= experimental
probability.

$$\frac{33}{120}$$

(1)

(b) During the **rest of the week**, Rosie will make 500 calls.

Using the results in the table, how many sales does she expect to make during the **rest of the week**?

$$\frac{6}{120} = \frac{1}{20}$$

1 out every 20 calls = Sale.

$$\frac{1}{20} \text{ of } 500 = 25 \text{ sales}$$

(2)
(Total 3 marks)

Q2.

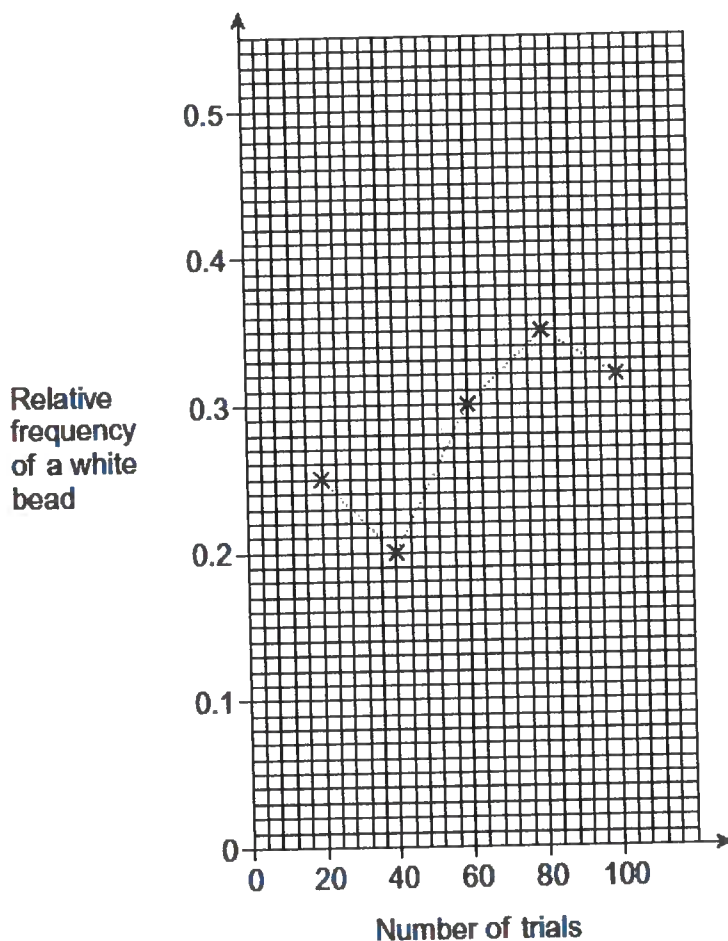A bag contains white beads, black beads and red beads.

The following trial is repeated 100 times.

> Pick a bead at random.
> Record the colour.
> Put the bead back in the bag.

The graph shows the relative frequency of a white bead after every 20 trials.

(a) Work out the number of times a white bead was picked in the first 20 trials.

$0.25 \times 20 = \underline{5}$

(2)

(b) What is the best estimate for the probability of picking a white bead?
Give a reason for your answer.

0.32 → This is the relative frequency for the greatest number of trials,

The more trials the more accurate!

(2)

(c)   There are a total of 1000 beads in the bag.

Estimate the number of beads that are white.

$0.32 \times 1000 = 320$

(2)
(Total 6 marks)

Q3.

A spinner was spun 200 times.
The relative frequency of landing on 4 after 50, 100, 150 and 200 spins is shown.

Number of spins	50	100	150	200
Relative frequency	0.14	0.13	0.18	0.16

(a)   Which relative frequency gives the best estimate of the probability of the spinner landing on 4? 0.16

Give a reason for your answer.

The more spins, the more accurate.

(2)

(b)   How many times did the spinner land on 4 from spin 51 to spin 100?

$0-50 \rightarrow 0.14 \times 50 = 7$ times

$0-100 \rightarrow 0.13 \times 100 = 13$ times.
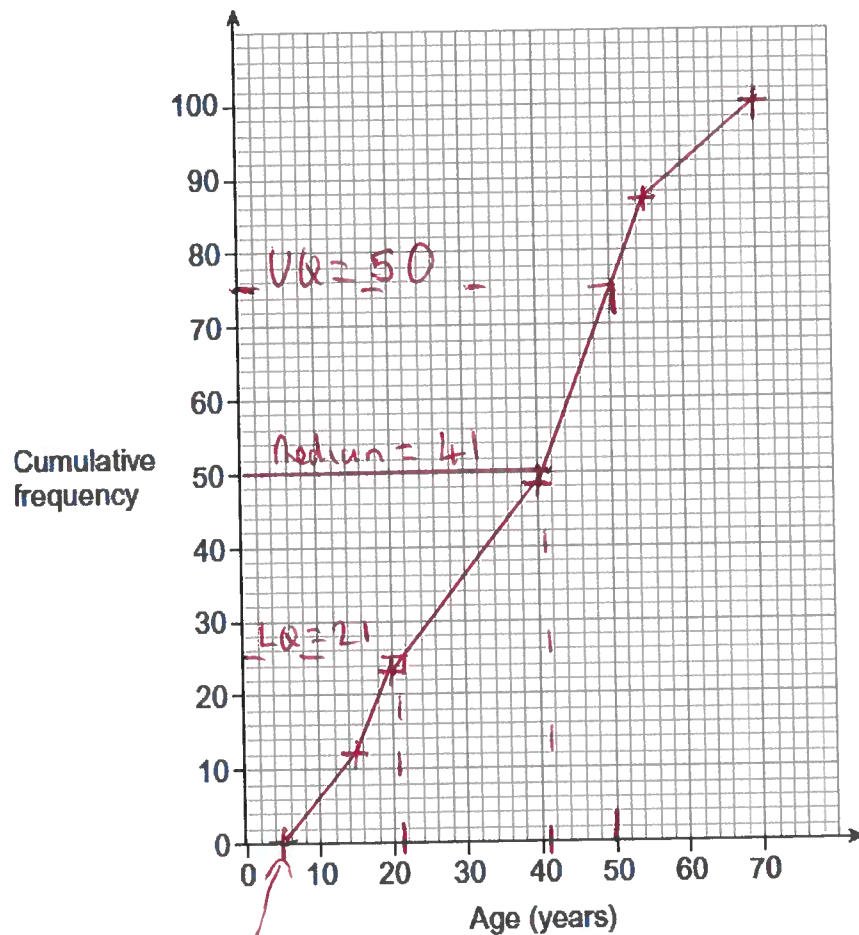
$13 - 7 = 6$ times

(3)
(Total 5 marks)

Q1.

The table shows information about the ages of 100 rugby supporters.

Age, a (years)	Frequency	c.f.
5 ≤ a < 15	12	12
15 ≤ a < 20	11	23
20 ≤ a < 40	25	48
40 ≤ a < 55	39	87
55 ≤ a < 70	13	100

(a)   Plot a cumulative frequency diagram for the data.

UQ = 50

Median = 41

LQ = 21

Cumulative frequency

Age (years)

(4)

(b)   The youngest supporter is 8 years old.
        The oldest supporter is 69 years old.

        Draw a box plot for the data.

Age (years)
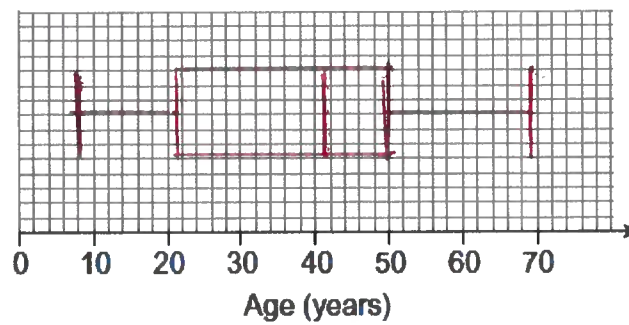
(3)
(Total 7 marks)

Q2.

The cumulative frequency graph shows information about the masses of 200 apples.

Mass, grams
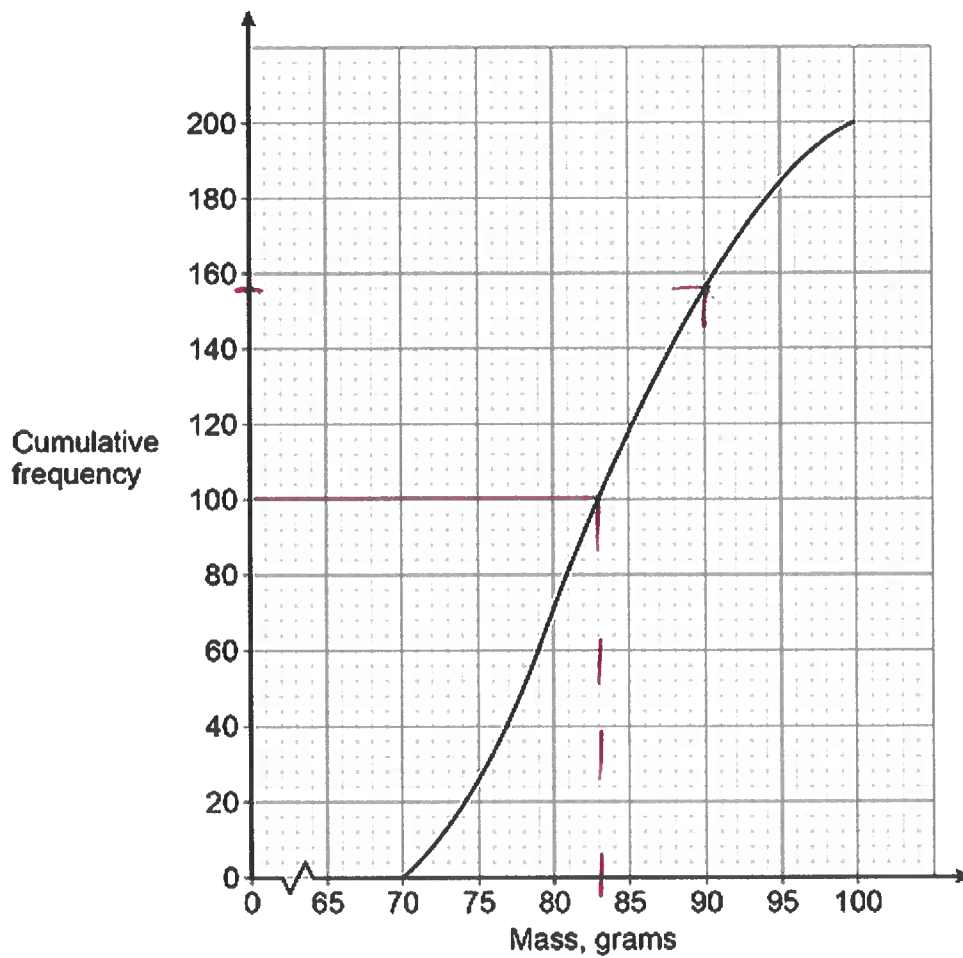
(a)   Estimate the median mass.   83

(1)

(b)   Apples with mass 90 grams or less cost 32p each.
       Apples with mass more than 90 grams cost 39p each.

       Estimate the **total** cost of the 200 apples.

$$156 \to < 90$$
$$44 \to > 90$$

$$156 \times 32 + 44 \times 39 = 6708p$$
$$= £67.08$$

(3)

(Total 4 marks)

Q3.

Two groups of people are trying to lose weight.

(a)   Group A join a gym.
       The graph shows information about their weight loss after one month.

Cumulative frequency

UQ = 2·6

Median = 1·2

LQ = 0·95

IQR = 1·65

Weight loss (kilograms)
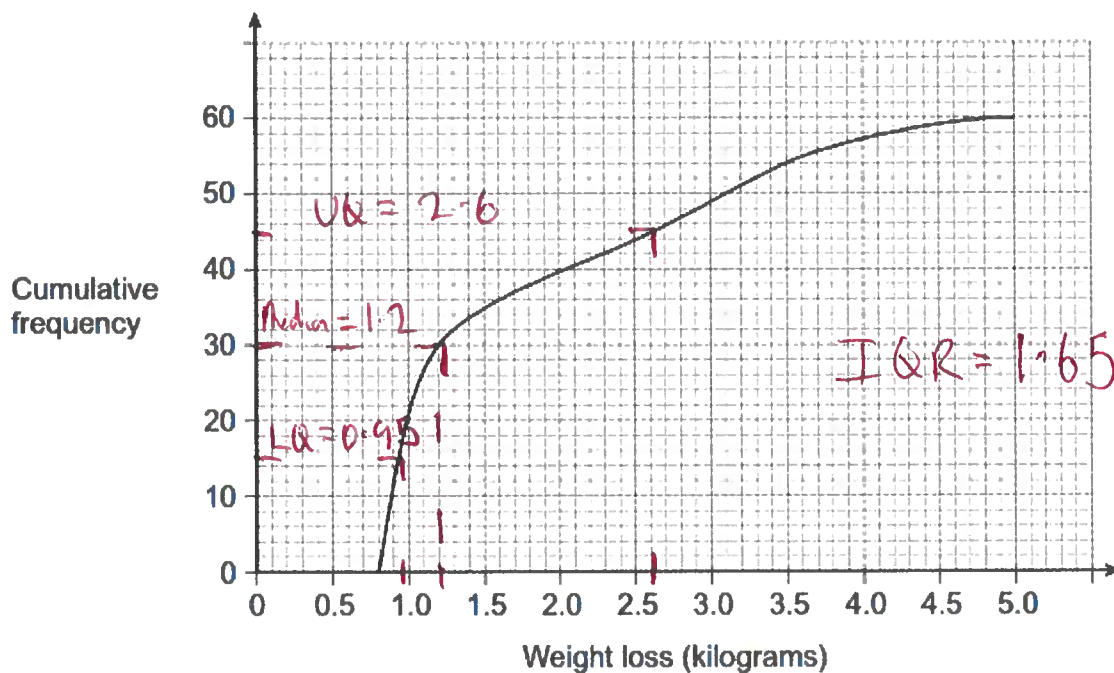
(i)   How many people are in group A?   60

(1)

(ii)   Does everyone in group A lose weight?   Yes.
       Write down how you decide.

       The horizontal axis starts at 0·8 kg lost

(1)

(b) Group B follow a diet.
The box plot shows information about their weight loss after one month.

Weight loss (kilograms)
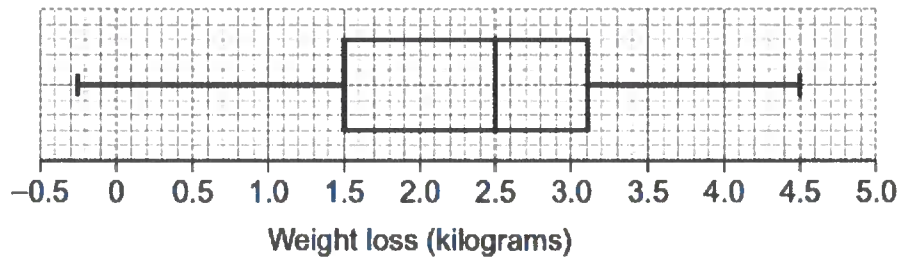
Does everyone in group B lose weight? *No*
Write down how you decide.

Someone puts on 0·25 kg !

(1)

(c) Compare the weight loss of group A with group B.

Group B lost more weight on average as their median is 2·5kg lost compared to Group A's median of 1·2kg lost.

Group A and B's Interquartile range were very similar (A = 1·65, B = 1·6) So the weight loss was equally consistent.

(5)

(Total 8 marks)

Q1.

Draw the graph of $y = 1 - \dfrac{1}{2}x$ for values of x from –2 to 4

y intercept = 1
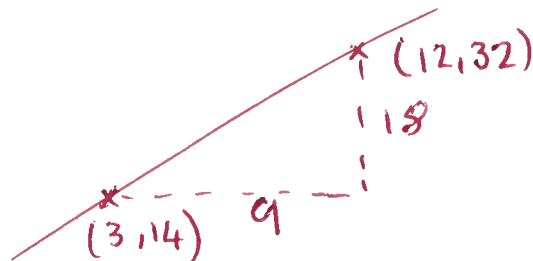gradient = $-\dfrac{1}{2}$

Or use
a table of
values.
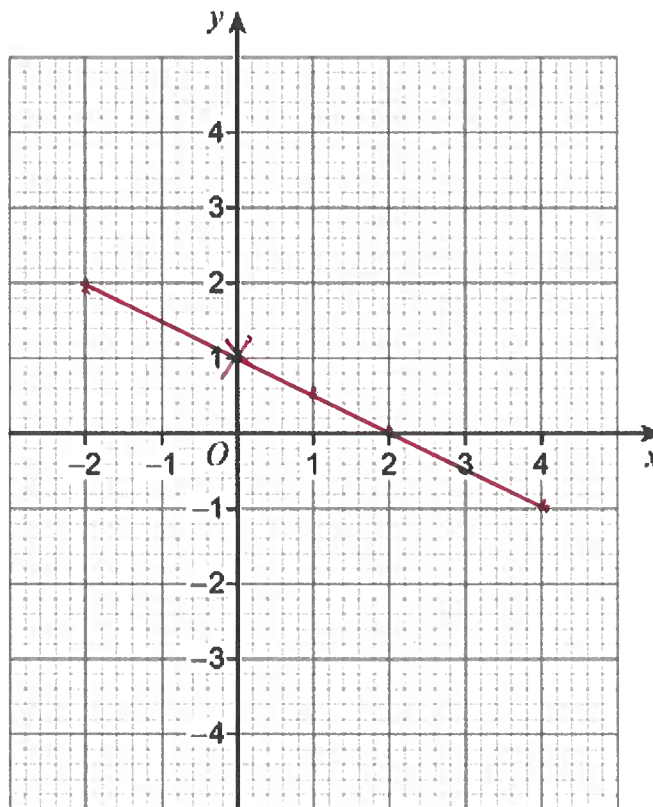
(Total 3 marks)

Q2.

A straight line passes through (3, 14) and (12, 32)

Work out the equation of the line.

Give your answer in the form $y = mx + c$

(12, 32)

18

(3, 14)   9

$$\text{Gradient} = \frac{32-14}{12-3} = \frac{18}{9} = 2$$

$$y = 2x + c$$

Sub in
$x = 3$
$y = 14$

$\begin{bmatrix} \text{or } x = 12 \\ y = 32 \end{bmatrix}$

$$14 = 2 \times 3 + c$$
$$14 = 6 + c$$
$$c = 8$$

$$y = 2x + 8$$

(Total 3 marks)

Q3.

Write down the equation of the straight line that

passes through the point (0, 4)

and

is parallel to the line $y = 5x + 3$

Parallel →
Same gradient

$$y = 5x + c$$

Sub in
$x = 0$
$y = 4$

$$4 = 5 \times 0 + c$$
$$4 = 0 + c$$
$$c = 4$$

$$y = 5x + 4$$

(Total 2 marks)

Likely – Simplifying algebraic expressions

Q1.

Expand $(x^2 - 9xy)(2x + 5y)$

	x^2	$-9xy$
$2x$	$2x^3$	$-18x^2y$
$+5y$	$+5x^2y$	$-45xy^2$

$$2x^3 - 13x^2y - 45xy^2$$

(Total 2 marks)

Q2.

Simplify $\dfrac{25a}{8} \times \dfrac{2a}{5}$

Give your answer as a single fraction in its simplest form.

$$\frac{25a}{8} \times \frac{2a}{5} = \frac{50a^2}{40} = \frac{5a^2}{4}$$

(Total 2 marks)

Q3.

Expand and simplify fully $4(2c + 3) - (5c - 1)$ ↙ Careful!

$$= 8c + 12 - 5c + 5$$
$$= 3c + 17$$

(Total 2 marks)

Q4.

Simplify fully $\dfrac{8x^2+4}{5x} \times \dfrac{3x}{14x^2+7}$

You **must** show your working.

$$= \frac{4(2x^2+1)}{5x} \times \frac{3x}{7(2x^2+1)} = \frac{12x(2x^2+1)}{35x(2x^2+1)}$$

$$= \frac{12}{35}$$

(Total 3 marks)

Likely – Congruence

Q1.

Which of these is **not** used to prove that triangles are congruent?

Circle your answer.

SSS   SAS   (AAA)   RHS
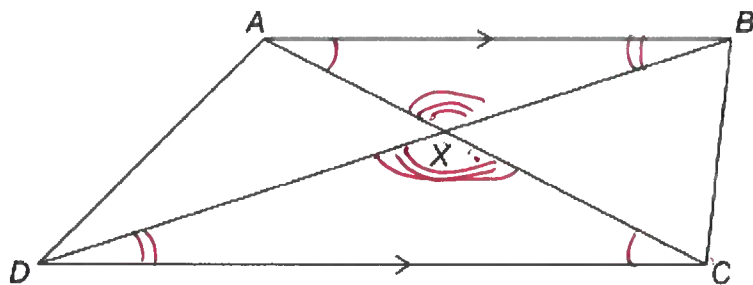
(Total 1 mark)

This only proves Similarity

Q2.

ABCD is a trapezium.

All four sides are different lengths.

AB is parallel to CD.

The diagonals intersect at X.

Not drawn accurately

For each statement, tick the correct box.

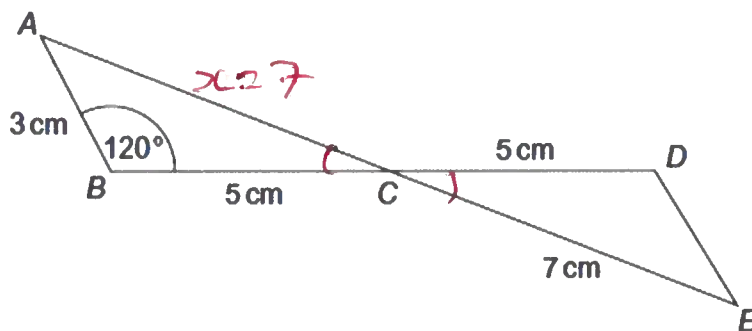|	True	May be true	Not true	
Triangles AXB and CXD are similar	✓			Angles all equal.
Triangles AXD and BXC are congruent			✓	
Angle ADB = angle BDC		✓		
Area of triangle ABC = area of triangle ABD	✓			

Base ~~perp~~ and perpendicular height are the same.

(Total 4 marks)

Q3.

The straight lines *AE* and *BD* intersect at *C*.

Not drawn accurately

A

3 cm

$x = 7$

120°

B

5 cm

5 cm

C

D

7 cm

E

Prove that triangles *ABC* and *EDC* are congruent.

Work out AC using Cosine rule.

$$x^2 = 3^2 + 5^2 - 2 \times 3 \times 5 \times \cos 120$$
$$= 49$$
$$x = 7$$

∴ Congruent SAS

AC = CE = 7cm
BC = CD = 5cm
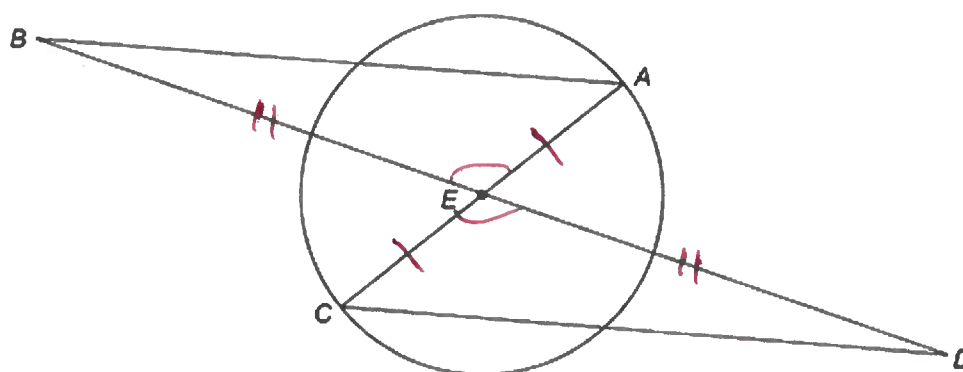$A\hat{C}B = D\hat{C}E$

(Total 5 marks)

vertically opposite.

Q4.

AC is a diameter of a circle, centre *E*.

E is the midpoint of *BD*.

B

A

E

C

D

Not drawn accurately

Prove that triangle *ABE* is congruent to triangle *CDE*.

AE = EC (both are radii)
BE = ED (E is midpoint)
$B\hat{E}A = C\hat{E}D$ (vertically Opposite)

∴ Congruent SAS

(Total 4 marks)

Likely – Expanding/factorising

Q1.

Expand $(x^2 - 9xy)(2x + 5y)$ See previous identical question!

(Total 2 marks)

Q2.

Factorise fully $x^3 - 49x$ $= x(x^2 - 49)$ ← Difference of 2 squares

$= x(x - 7)(x + 7)$

(Total 2 marks)

Q3.

Factorise $x^2 + 2x - 24$ $= (x + 6)(x - 4)$

(Total 2 marks)

Q4.

Factorise $25a^2 - b^2$ $= (5a + b)(5a - b)$

(Difference of 2 squares)

(Total 1 mark)

Q5.

Expand and simplify fully $(x - 3)(x - 4)(x + 8)$

	x	-3
x	x^2	$3x$
-4	$-4x$	$+12$

$= x^2 - 7x + 12$

	x^2	$-7x$	$+12$
x	x^3	$-7x^2$	$+12x$
$+8$	$+8x^2$	$-56x$	$+96$

$= x^3 + x^2 - 44x + 96$

(Total 3 marks)

Q6.

Factorise fully $144 - 4x^2$ $= (12 - 2x)(12 + 2x)$

$12^2 - (2x)^2$

(Diff of 2 squares)

(Total 2 marks)

Q7.

(a) Factorise $5x^2 + 6x - 8$

$5x \cdot -8 = -40 = -4 \times 10$
$6 = -4 + 10$

	$5x$	-4
x	$5x^2$	$-4x$
$+2$	$+10x$	-8

$(5x - 4)(x + 2)$

(2)

(b) Simplify fully $\dfrac{x^2 + 9x + 14}{x^2 - 4}$ $= \dfrac{(x+2)(x+7)}{(x+2)(x-2)}$

$= \dfrac{x+7}{x-2}$

(3)

(Total 5 marks)

Likely – Solve linear equations

Q1.

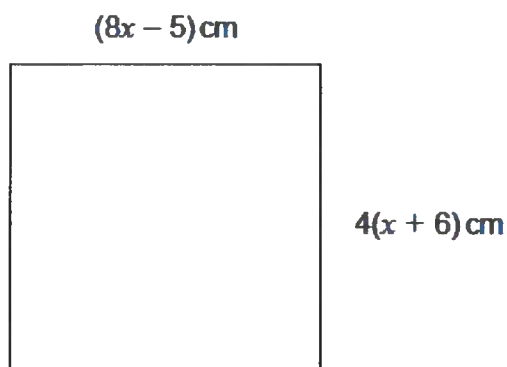Solve $5x + 11 = 3x + 19$

$$5x + 11 = 3x + 19$$

$-3x \quad 2x + 11 = 19 \quad -3x$

$-11 \quad 2x = 8 \quad -11$

$\div 2 \quad x = 4 \quad \div 2$

(Total 2 marks)

Q2.

The diagram shows a square.

$(8x - 5)$ cm

$4(x + 6)$ cm

Work out the length of one side of the square. → All sides equal

$$8x - 5 = 4(x + 6)$$

$-4x$ | $8x - 5 = 4x + 24$ | $-4x$
$+5$ | $4x - 5 = 24$
$\div 4$ | $4x = 29$
| $x = \dfrac{29}{4}$ | $\div 4$

Sub into either expression $8 \times \dfrac{29}{4} - 5 = 53$

(Total 4 marks)

Q3.

Solve

$$\frac{x + 15}{3} = 2(x + 10)$$

$\times 3$ | $x + 15 = 6(x + 10)$ | $\times 3$
$-x$ | $x + 15 = 6x + 60$ | $-x$
| $15 = 5x + 60$ | -60
-60 | $-45 = 5x$ | $\div 5$
$\div 5$ | $-9 = x$

(Total 4 marks)

Q4.

Work out the value of x when

$x - 20 : x + 280$ simplifies to $1 : 4$

$(x - 20) \times 4 = x + 280$

$4x - 80 = x + 280$

$-x$ $3x - 80 = 280$ $-x$

$+80$ $3x = 360$ $+80$

$\div 3$ $x = 120$ $\div 3$

$\left[\begin{array}{l} x - 20 : x + 280 \\ = 100 : 400 = 1 : 4 \end{array} \right]$ **(Total 4 marks)**

Likely – Averages and range

Q1.

Five integers have:

     a mode of 1

     a median of 2

     a mean of 3 $\rightarrow$ Total $= 5 \times 3 = 15$

What is the greatest possible range of the five integers?

You **must** show your working.
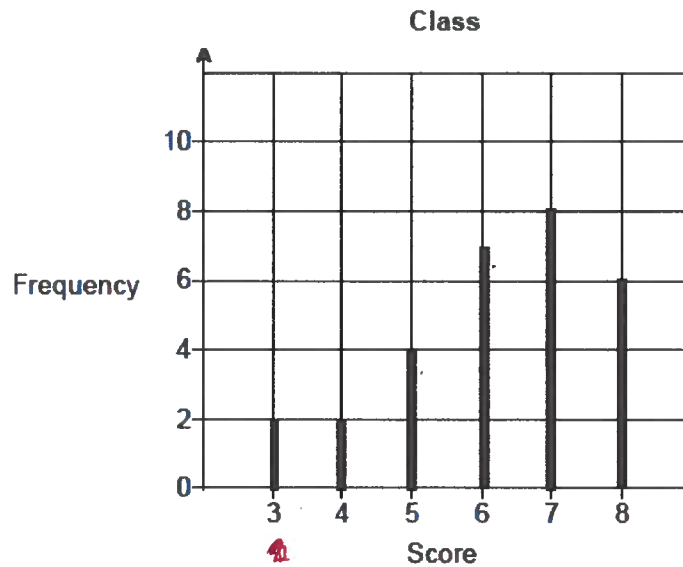
$1 \quad 1 \quad 2 \quad 3 \quad 8$

Must add to be 11
One = 3
other = 8   **(Total 3 marks)**

Max Range = 7

Q2.

Students in a class took a spelling test.

The diagram shows information about the scores.

Class

Lucy is one of the 29 students in the class.

Her score was the same as the **median** score for her class.

Work out her score. **6**

$$\frac{2a+1}{2} = 16^{th} \text{ person}$$
$$\text{is median}$$

3 3 4 4 5 5 5 5 6 6 6 6 6 6 — · · · · · ·
                    ↑
                   16th

(Total 2 marks)

Q3.

The number of goals scored by 20 players in a season is shown.

Number of goals	Frequency	Midpoint	f × mp
0 to 4	6	2	12
5 to 9	11	7	77
10 to 14	3	12	36
	Total = 20		125

Work out an estimate of the mean number of goals per player.
Give your answer as a decimal.

125 ÷ 20 = 6·25

(Total 3 marks)

Likely – Venn diagrams

Q1.

A school has 86 teachers.

42 are male and 44 are female.
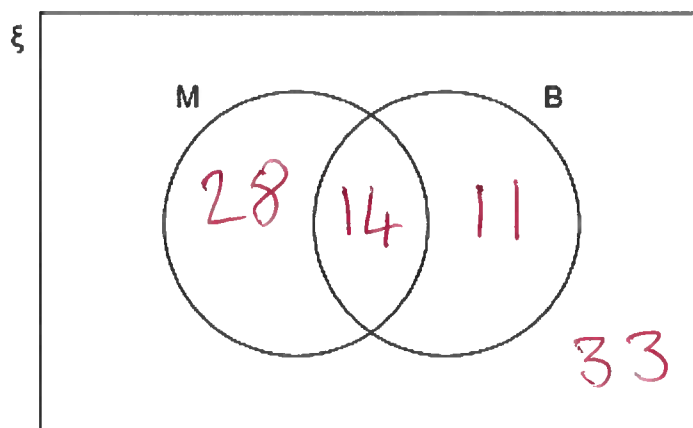
$\frac{1}{3}$ of the male teachers have blue eyes. $\frac{1}{3}$ of $42 = 14$

$\frac{1}{4}$ of the female teachers have blue eyes. $\frac{1}{4}$ of $44 = 11$ $\Big\}$ 25 blue eyes

(a) ξ = teachers in the school

M = male teachers

B = teachers who have blue eyes

Complete the Venn diagram.

(3)

(b) One teacher who has blue eyes is chosen at random. → Out of 25

Work out the probability that the teacher is male.

$$\frac{14}{25}$$
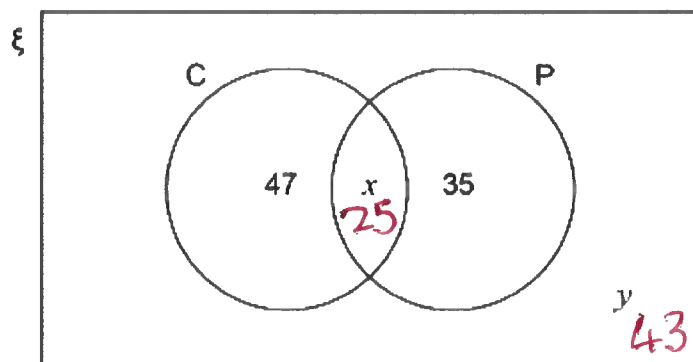
(1)
(Total 4 marks)

Q2.

The Venn diagram shows some information about 150 students.

ξ = 150 students

C = students who study Chemistry

P = students who study Physics

The probability that a Physics student, chosen at random, also studies Chemistry is $\frac{5}{12}$

One of the 150 students is chosen at random.

Work out the probability that the student does **not** study either Chemistry or Physics.

$\downarrow$

$\times 12 \quad \dfrac{x}{35+x} = \dfrac{5}{12} \quad \times 12$

$\times (35+x) \quad \dfrac{12x}{35+x} = 5 \quad \times (35+x)$

$12x = 5(35+x) \quad \text{expand}$

$12x = 175 + 5x \quad -5x$

$-5x \quad 7x = 175$

$\div 7 \quad x = 25 \quad \div 7$
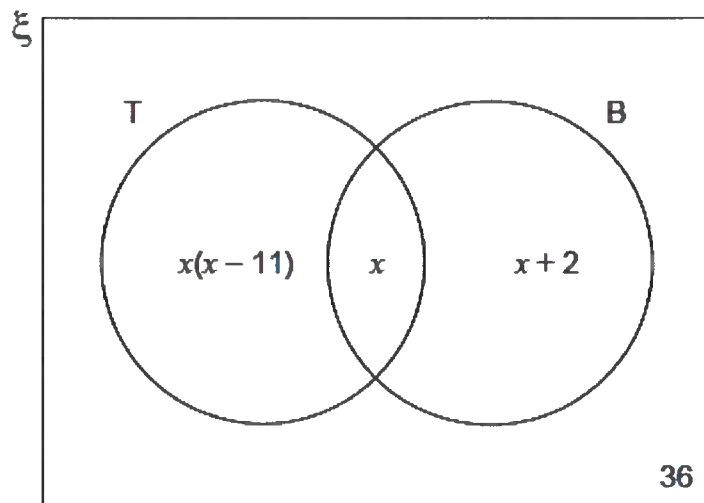
$\dfrac{43}{150}$

(Total 4 marks)

Q3.

The Venn diagram shows information about a coin collection.

ξ = 150 coins in the collection

T = coins from the 20th century

B = British coins

A coin is chosen at random.

It is British.

Work out the probability that it is from the 20th century.

$x(x - 11) + x + x + 2 + 36 = 150$

$x^2 - 11x + x + x + 38 = 150$

$x^2 - 9x + 38 = 150$

$x^2 - 9x - 112 = 0$

$(x + 7)(x - 16) = 0$

$x = -7 \qquad x = 16$

Doesn't make sense

$16(16 - 11) = 80$

$80 + 16 = 96$

$\dfrac{96}{150}$

(Total 5 marks)